Migration without migration noise

Peter W. Cary, Sensor Geophysical Ltd. & The CREWES Project

It is now well recognized (Gray, 1992; Lumley et al., 1994) that antialiasing of Radon integration operators, such as Kirchhoff migration, is required once the slope of the line or surface of integration exceeds the aliasing slope of the data, as determined by the sample rates in space and time. The operator antialiasing criterion for Radon transforms of poststack data can be derived as a consequence of assuming that the function within the Radon integral is bandlimited. Applying this same assumption, in its appropriate form, to other types of Radon integrals yields formulas that, for regularly sampled datasets, can be used to compute 2-D and 3-D, prestack and poststack migrated images that, except for edge effects, are free of migration noise (operator aliasing artifacts). Although the results stated here assume regular sampling of the dataset, regular sampling is not crucial to the analysis since in principle it is possible to construct a regularly sampled bandlimited function from its irregular samples, as long as the average sample rate is above the Nyquist rate (e.g. Cary, 1997).

In a simplified form, 2-D generalized Radon transforms are all of the form

$$U(\tau,p)=\iint u(t,x)\delta(t-t'(\tau,p,x))dtdx,$$

where, for example, $t'=\tau+px$ for slant stacks and $t'=(\tau^2+4(p-x)^2/v^2)^{1/2}$ for poststack migration. Aliasing artifacts are generated when the following commonly used approximation to the Radon integral is used:

$$U(\tau,p)=\sum_l\sum_j u(j\Delta t,l\Delta x)Sinc(\tfrac{t'}{\Delta t}-j)$$

(see Marfurt et al., 1996, for examples). Hale (1984) was able to derive an expression for eliminating operator aliasing artifacts using the fact that $u(t,x)$ is bandlimited in both time and space with sample rates, Δt and Δx, which are both small enough to prevent aliasing. In this case the continuous wavefield, $u(t,x)$, can be exactly reconstructed from its discrete samples with sinc function interpolation:

$$u(t,x)=\sum_l\sum_j u(j\Delta t,l\Delta x)Sinc(\tfrac{t}{\Delta t}-j)Sinc(\tfrac{x}{\Delta x}-l),$$

where $Sinc(x)=\sin(\pi x)/\pi x$. Substituting this expression into the Radon transform integral, we get

$$U(\tau,p)=\sum_l\sum_j u(j\Delta t,l\Delta x)\int Sinc(\tfrac{t'}{\Delta t}-j)Sinc(\tfrac{x}{\Delta x}-l)dx,$$

which can be evaluated to yield

$$U(\tau,p)=\sum_l\sum_j u(j\Delta t,l\Delta x)q(l\Delta x)Sinc\{q(l\Delta x)(\tfrac{t'}{\Delta t}-j)\},$$

where $q(l\Delta x)=\min(1,\frac{\Delta t}{\Delta x}\left|\frac{dt'}{dx}\right|^{-1})$. The only assumption needed to evaluate the integral over x is that the line of integration varies slowly enough that $t'(x)\cong t'(l\Delta x)+(x-l\Delta x)\frac{dt'}{dx}$. The $q(l\Delta x)$ factors in the final summation antialias the summation operator by removing the aliased frequencies from the interpolating sinc function.

When evaluating a Radon integral of prestack data, like 2D common-shot prestack migration, which is of the form:

$$U(\tau,x)=\iint u(t,x,r)\delta(t-t'(\tau,x,r))dtdr,$$

where x is the output location, r is receiver position, and t' is the traveltime from source to image point to receiver, we need to keep in mind that Δr, the receiver interval, was chosen small enough at the design stage to prevent aliasing of events that dip with respect to horizontal reflections (Stone, 1994). Since NMO is all that is needed to "migrate" a hyperbola recorded from a horizontal reflection, we should use sinc interpolation of the NMO-corrected data, u_{NMO}, not the recorded data, u, to evaluate prestack Radon integrals. The appropriate formula is therefore

$$u(t+t_{NMO},r) = u_{NMO}(t,r) = \Delta r \sum_l \sum_j u_{NMO}(j\Delta t, l\Delta r) Sinc(\tfrac{t}{\Delta t} - j) Sinc(\tfrac{r}{\Delta r} - l),$$

which states that the continuous NMO-corrected wavefield from one shot can be reconstructed from the discretely sampled traces in the gather. Upon substitution into the Radon integral, this leads to the following complete (except for edge effects) one-fold image of the subsurface:

$$U(\tau,x) = \Delta r \sum_l \sum_t u_{NMO}(j\Delta t, l\Delta r) q(\Delta t, l\Delta r) Sinc\{q(\Delta t, l\Delta r)(\tfrac{t'(\tau,r)-t_{NMO}}{\Delta t} - j)\},$$

where $q(\Delta t, l\Delta r) = \min(1, \frac{\Delta t}{\Delta r}\left|\frac{d(t'-t_{NMO})}{dr}\right|^{-1})$. One-fold images from other shot gathers will improve the signal-to-noise ratio of the final result, but not the fidelity with which the signal is imaged. This formula says that, for prestack migration, operator antialiasing should be applied when the difference in slope between the line of integration and the hyperbola corresponding to a horizontal reflector becomes greater than the aliasing slope, $\Delta t/\Delta x$. Consequently, even if flat events are aliased on un-NMO corrected data, they will still migrate correctly (without any lowering of frequency content other than NMO-stretch) whether they are imaged using NMO+stack+poststack migration or using prestack time migration.

Operator antialiasing conditions can also be derived for a 3-D Radon transform such as 3-D poststack migration:

$$U(\tau, p_x, p_y) = \iiint u(t,x,y)\delta(t - t'(\tau,p,q,x,y)dtdxdy,$$

where p_x, p_q are the output inline, crossline positions, x, y are the input inline and crossline positions, and t' is the traveltime from source to image point to receiver. Using sinc functions to interpolate the stack image $u(t,x,y)$,

$$u(t,x,y) = \sum_k \sum_l \sum_j u(j\Delta t, l\Delta x, k\Delta y) Sinc(\tfrac{t}{\Delta t} - j) Sinc(\tfrac{x}{\Delta x} - l) Sinc(\tfrac{y}{\Delta y} - k),$$

yields the following 3-D migrated image that is free of operator aliasing artifacts:

$$U(\tau, p_x, p_y) = \Delta x \Delta y \sum_k \sum_l \sum_j u(j\Delta t, k\Delta x, l\Delta y) q(k\Delta x, l\Delta y) Sinc\{q(k\Delta x, l\Delta y)(\tfrac{t'(\tau, l\Delta x, k\Delta y)}{\Delta t} - j)$$

where $q(k\Delta x, l\Delta y) = \min(1, \frac{\Delta t}{\Delta x}\left|\frac{dt'}{dx}\right|^{-1}, \frac{\Delta t}{\Delta y}\left|\frac{dt'}{dy}\right|^{-1})$. Notice that this derivation yields a 3-D operator antialiasing criterion that depends only on spatial sample rates in the x and y directions, not on the effective spatial sample rate in the ray direction, as used by Lumley et al. (1994). As a result of merging this result with the previous result from 2-D prestack migration, it is evident that only cross-spreads from land 3-D prestack datasets (i.e. pairs of orthogonal shot and receiver lines) will be capable of providing (except for edge effects) complete one-fold images of the subsurface since cross-spreads are typically the only subsets of the full 3-D dataset that have inline and crossline sample rates that correspond to the Nyquist sample rates of the NMO-corrected wavefield. So as far as imaging signal is concerned, the 3-D cross-spread is an ideal acquisition configuration. However, since spatial sample rates in seismic surveys need to satisfy requirements of coherent noise rejection as well as signal imaging criteria, designing 3-D surveys based on the ability to image cross-spreads alone is probably not a good idea.

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