Some basic imaging problems with regularly-sampled seismic data

Peter W. Cary and Xinxiang Li, Sensor Geophysical Ltd.*

Summary

The fact that irregular spatial sampling of 3D surveys can lead to processing artifacts has been recognized for some time (Gardner and Canning, 1994), and methods such as least-squares DMO, azimuth moveout and migration are beginning to be used to overcome these problems (Ronen and Liner, 2000). It is often assumed that these sophisticated inversion methods are required only when spatial sampling is irregular, and that regularly-sampled seismic surveys do not suffer from sampling problems. However, this is not necessarily true.

The purpose of this paper is to illustrate in a tutorial manner the potential prestack sampling problems that can exist in the simple situation of a 2-D regularly-sampled dataset acquired over a constant-velocity, isotropic model. Prestack sampling problems are shown to follow from the method that we use for selecting the CMP bin size from poststack (zero-offset) data. Partial stacking into pseudo-common offset sections is a simple method that we have used for decades to overcome this sampling problem (whether we were aware of it or not). Although partial stacking has obvious limitations, it deserves respect as a legitimate least-squares solution to the problem. A simple method of 2-D offset continuation is presented here, which offers some improvement over partial stacking.

Prestack sampling problems from zero-offset design

We begin by showing that the criterion for choosing the CMP bin size of seismic surveys (small enough to avoid data aliasing at the shallowest target on a zero-offset section (Liner and Underwood, 1999)) implies that all the prestack data gathers could suffer from data aliasing.

Consider the case of a single dipping reflector of dip a, within a medium of constant velocity v such as in Figure 1.

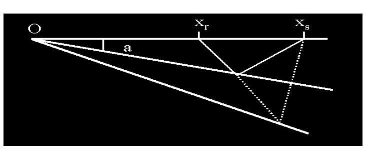

Fig. 1 Geometry of dipping reflector.

On a zero-offset section the highest unaliased frequency is well known to be

$$f_m = v / 4\, \Delta x_m \sin a, \qquad (1)$$

where Δx_m is the CMP sampling interval. The highest unaliased frequency on a shot gather is given by

$$f_s = v^2 t / 2\, \Delta x_r\, (x_r - x_s \cos 2a), \qquad (2)$$

where $\Delta x_r = 2\Delta x_m$ is the receiver interval, and x_r and x_s are the receiver and source positions with respect to the intersection of the dipping plane and the surface. This equation follows from analysis of the slope of the traveltime curve, which is given directly by the cosine rule.

If f_s is ever less than f_m for any choice of coordinates and dip angle, then aliased frequencies can exist on a shot gather even though the same frequencies on a zero-offset section will not be aliased. This situation will arise whenever

$$v\, t \sin a / (x_r - x_s \cos 2a) < 1, \qquad (3)$$

which is a condition that can be satisfied without too much difficulty. Figure 2 shows a synthetic zero-offset section with four events dipping at 0°, 15°, 30° and 45° in a medium with a constant velocity of 2000 m/s. The F-K amplitude spectrum on the right shows that the events are not aliased.

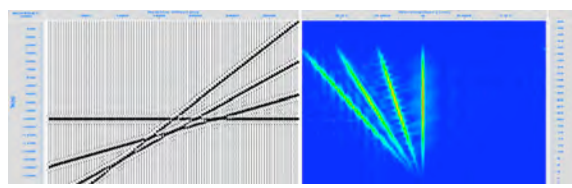

Fig. 2 Unaliased events on a zero-offset section with F-K spectrum

Figure 3 shows a portion of one shot gather as recorded over these four reflectiors. The F-K spectrum clearly shows that the high frequencies on the steepest events are aliased on the shot gather even though they are not aliased on the zero-offset section.

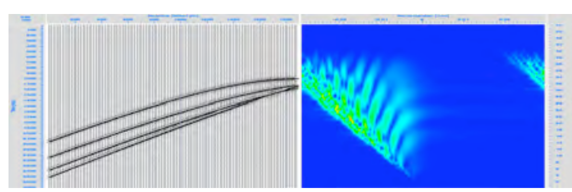

Fig. 3 The same events are aliased on a shot gather.

Since it is often assumed that shot records can be individually prestack migrated, this potential aliasing condition is obviously a concern. However, it is possible to migrate aliased data, and get close to the correct answer by stacking aliased results, which will be illustrated in the following sections with aliased common-offset sections.

Of course, the actual CMP interval that we use in practice may be smaller than specified by Eqn. (1), so that data aliasing on shot gathers never occurs. A worse case is if the CMP interval is too large, since the highest frequencies may not survive the processing sequence to reach the final image. Regardless of the real situation, it is worthwhile to know that when the CMP interval is chosen in accordance with Eq. (1), we can expect to encounter prestack sampling problems. Prestack imaging steps need to take this into account.

The simplest least-squares solution: partial stacking

In the previous section it was stated that all prestack gathers could suffer from data aliasing when the CMP interval is chosen according to Eqn. (1), but the problem was only demonstrated with shot gathers. Since shot intervals are typically greater than or equal to receiver intervals, it takes no further analysis to realize that data aliasing on receiver gathers will usually be worse than on shot gathers.

Common-offset sections will also be aliased, unless the shot interval is half the receiver interval, because the spatial sample interval within each common-offset section is the shot interval (assuming regular sampling). Figure 4 illustrates the data aliasing that occurs on a common-offset section when the shot interval equals the receiver interval.

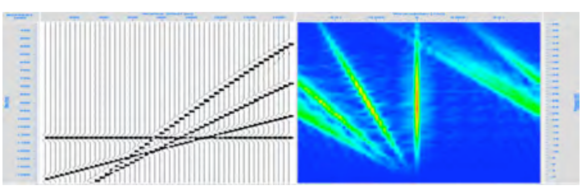

Fig. 4 An aliased true common-offset section.

The use of the term "common-offset section" (COS) in the previous paragraph requires some clarification since it can lead to some confusion. A very common use of this same term, or other terms such as "common-offset panel" is to describe the collection of traces with close to the same offset (within an offset bin width), such that the collection of traces when taken together, or partially stacked, have at least one trace occupying each CMP location. An offset bin width equal to twice the shot interval yields one trace per CMP location within each pseudo-COS when the sampling is perfectly regular. Pseudo-COS's are used so often in processing that they are usually just called common-offset sections or panels without much care being taken to distinguish them from true COS's.

One of the main reasons that a pseudo-COS is so useful is that it overcomes the prestack aliasing problem of true-COS's so well, at least when the shot move-up is small. Figure 5 illustrates that partial stacking almost completely solves the aliasing problem when the shot interval equals the receiver interval.

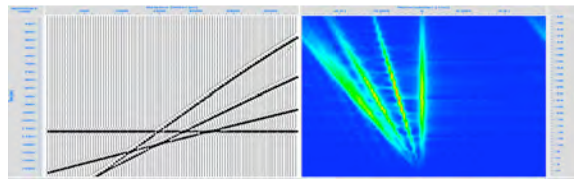

Fig. 5 An unaliased pseudo-common-offset section.

The same basic principles that least-squares DMO and migration use to overcome aliasing due to irregular sampling are being used by partial stacking to overcome the aliasing problem due to regular sampling. The interleaving of two true-COS's to form the pseudo-COS in Figure 5 is an example of the generalized form of sampling (Papoulis, 1977) that Ronen (1987) used to illustrate his initial formulation of least-squares DMO.

Unlike DMO or offset continuation, partial stacking assumes that the data can simply be modelled as being constant over a small range of offsets, after partial NMO to the offset at the center of each offset bin. So the very simple forward modeling equations are:

$$\mathbf{d} = \mathbf{L}\, m \qquad (4)$$

where $\mathbf{d} = (d_1, d_2, \ldots, d_N)^T$ is a vector of the input samples that fall within the offset bin at a particular CMP and time after partial NMO, m is the single model parameter to solve for at that CMP and time, and $\mathbf{L} = (1, 1, \ldots, 1)^T$. The least-squares solution is

$$m = (\mathbf{L}^T \mathbf{L})^{-1} \mathbf{L}^T \mathbf{d} = \mathbf{N}^{-1} \sum d_i\,, \qquad (5)$$

which is simply the average of the data within the offset bin, i.e. the partial stack. Notice that since the problem is completely overdetermined, the solution is unique. No additional model constraints such as smoothness or sparseness are required, unlike least-squares DMO or migration.

The limitations of partial stacking

The previous example showed how well partial stacking overcomes the aliasing of regularly sampled data when the shot interval is small. However, the accuracy of this technique must obviously decrease as the shot interval increases. The following example illustrates the limitations of partial stacking.

The same set of four dipping reflectors in a 2000m/s medium are used as before, but now the geometry of the survey is taken from a real 2-D land survey where the receiver interval is 34m and the shot interval is 5 times larger (170m). As with most land lines, most of the geometry is regular, but there are gaps in the shooting pattern, and skidded shots that try to make up for the gaps.

Figure 6 shows a severely aliased, true 1000m common-offset section. Since the shot move-up is usually 5 stations, there is nominally one trace for every 10 CMP locations.

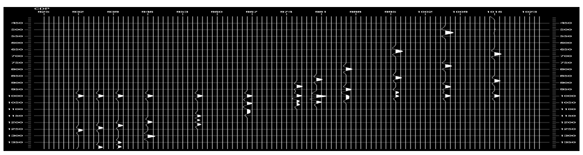

Fig.6 A severely aliased true common-offset section

In this case, at least 10 true COS's need to be interleaved to create a pseudo-COS that has at least one trace per CMP location. Figure 7 shows the partial stack of the pseudo-COS with offsets of 1000 ± 170m.

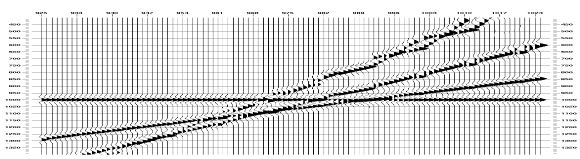

Fig. 7 The partial stack of a pseudo-common-offset section.

Both positive and negative offsets have been stacked together in order to fill some of the gaps due to missing or skidded shots. This is what causes the braided appearance of the steeper events in Figure 7.

This example uses a small velocity (2000m/s) and shallow times (less than 1.5s), to illustrate the limitations of partial stacking clearly. Applying partial NMO-correction to the bin center obviously does not move the dipping events to the correct position. The data should be offset continued, rather than NMO-corrected, to the bin center. Notice, however, that partial NMO still images the flat event perfectly.

Prestack migration of aliased data

Prestack migration is often performed with the Kirchhoff method, especially for wide-azimuth 3-D surveys, simply because it can honor the true irregular geometry of the input traces. Essentially, each trace is migrated separately and then added to the output buffer in the hope that everything adds up correctly in the end. We now test how well this procedure works on the synthetic example.

Figure 8 shows the prestack migration of the severely aliased, true 1000m common-offset section in Figure 6.

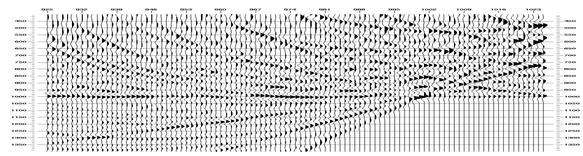

Fig. 8 Prestack migration of a true common-offset section.

This result is obviously dominated by artifacts, but if we now add together all the prestack migrated traces with positive and negative offsets in the range 1000 ± 170m, then we get the much better looking result in Figure 9. The events have moved to the correct locations, but notice that artifacts exist because of the regular and irregular spacing of the input traces.

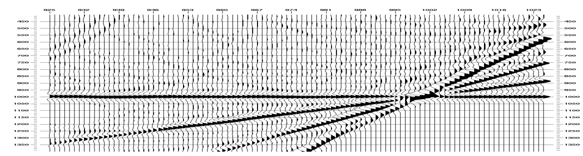

Fig. 9 Prestack migration of a pseudo-common-offset gather.

An alternative to prestack migrating each trace separately is to prestack migrate after least-squares regularization (with partial stack). In this case, the migration assumes that the true offset of all traces in Figure 7 is 1000m. The result is shown in Figure 10.

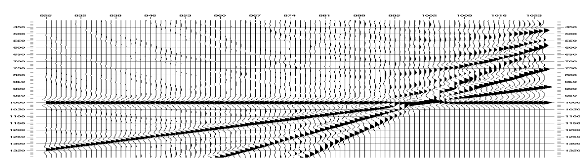

Fig. 10 Prestack migration of partially stacked data in Fig. 7

A quick comparison of Figures 9 and 10 would indicate that Figure 9 is far better. However, upon closer inspection, it can be seen that the waveforms of the flatter events are far better in Figure 10, as shown by the close-ups in Figures 11 and 12.

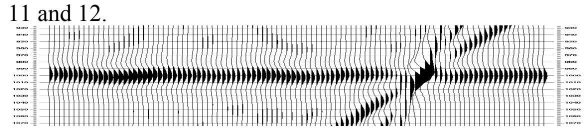

Fig. 11 Close-up of flat event in Fig. 9.

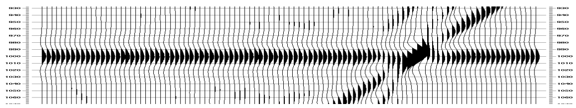

Fig. 12 Close-up of flat event in Fig. 10.

The geology where this survey was acquired consists of flat-lying reflectors, so on the real data prestack migration after partially stacking the data first gave better waveforms, as illustrated in the middle part of Figure 13.

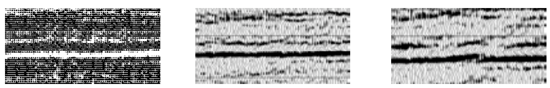

Fig. 13. Left: Partial stack of input traces. Middle: Migration of partial stack. Right: Prestack migration without partial stack.

Of course, when the geology is completely flat-lying, there is little point to using prestack migration. Notice that NMO + full stack is a pseudo-COS where the offset bin size is made large enough to include all offsets. Therefore, NMO + stack is our simplest least-squares regularization process, and works extremely well at preserving the waveforms on flat-lying events.

A simple method of offset continuation

Notice that we could have chosen to stack the data in Figure 7 after applying an NMO velocity that is correct for the steepest event, but wrong for the other reflectors, as in Figure 14.

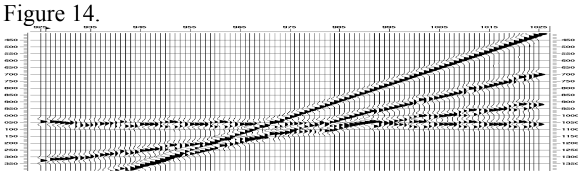

Fig. 14. Partial stack with v / cos(45°)

If we apply an F-K filter to select out the correctly imaged slopes after forming partial stacks for a sequence of different reflector dips, then we are performing offset continuation by dip-decomposition, along the same lines as the DMO method of Jacubowicz (1990). Offset continuation with this method is simple, efficient and accurate. Each partial dip-stack is the solution of a separate least-squares problem, but spatial aliasing can still cause the leakage of energy from one partial dip-stack to another.

Figure 15 shows the result of offset-continuation applied to the same pseudo-COS as was used to form Figure 7.

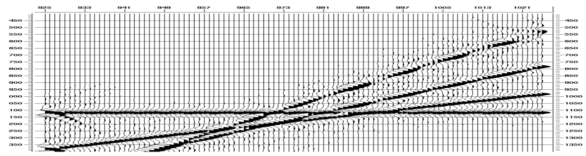

Fig. 15 Offset continuation of a pseudo-common-offset section.

The steep events have now moved to the correct location, but the amplitudes are still irregular. This is caused by the fact that offset continuation has moved small offsets down-dip and large offsets up-dip. A smoothness constraint needs to be included to force a more desirable solution with this limited amount of data. A complete least-squares solution with an offset-continuation operator would include several iterations of a procedure that attempts to minimize the misfit between the modeled and original data, as well as imposing an appropriate model constraint at each step. Figure 15 would be the zeroth iteration in such a procedure.

Conclusions

Regularly-sampled seismic surveys can suffer from spatial aliasing problems as much as irregularly-sampled surveys. Partial stacking into common-offset sections is the simplest least-squares solution to the problem. More sophisticated operations such as offset continuation can be used to regularize regularly-sampled data into COS's before prestack migration as well as irregularly-sampled data.

References

Gardner, G.H.F and Canning, A., 1994, Effects of irregular sampling on 3-D prestack migration: 64th Ann. Internat. Mtg., Soc. Expl. Geophys., Expand. Abstracts, 1553-1556.

Jacubowicz, H., 1990, A simple efficient method of dip-moveout correction: Geophys. Pros., **38**, 221-245.

Liner, C. L. and Underwood, W., 1999, 3-D seismic survey design for linear v(z) media: Geophysics, **64**, 486-493.

Papoulis, A., 1977, Generalized sampling expansion: IEEE, Trans. Circuit Syst., **CAS 24**, 652-654.

Ronen, J., 1987, Wave-equation trace interpolation: Geophysics, **52**, 973-984.

Ronen, S. and Liner, C.L., 2000, Least-squares DMO and migration: Geophysics, **65**, 1364-1371.

Acknowledgements

We would like to thank the sponsors of the CREWES Project, U. of Calgary, where much of this work was done.